Minitest 1 - MTH 2410

Dr. Graham-Squire, Fall 2013

≈ 15 min.

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first five questions of the test, however you should still show all of your work. No calculators are allowed on the last two questions.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 7. Total Points = 40.

1. (8 points) TRUE OR FALSE. Circle the correct answer. If true, explain (briefly) why it is true. If false, give a counterexample or explain (briefly) why it is false.

(a) **True or** (False:) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ and $\mathbf{u} \neq \mathbf{0}$, then $\mathbf{v} = \mathbf{w}$.

✓ False $\vec{u} = \langle 1, -1 \rangle$ ✓

$\vec{v} = \langle 2, 2 \rangle$ Then $\vec{u} \cdot \vec{v} = 0$

$\vec{u} \cdot \vec{w} = 0$

$\vec{w} = \langle 3, 3 \rangle$ but $\vec{v} \neq \vec{w}$

(b) **True or** (False) If $\mathbf{u} = \langle 1, -3 \rangle$ and $\mathbf{v} = \langle -2, 5 \rangle$, then $\mathbf{u} \times \vec{\mathbf{v}}$ will have only negative entries.

False. Cannot cross vectors in 2-D. ✓

(c) **True or** (False:) If plane P and plane Q are both orthogonal to plane R, then P and Q must be parallel.

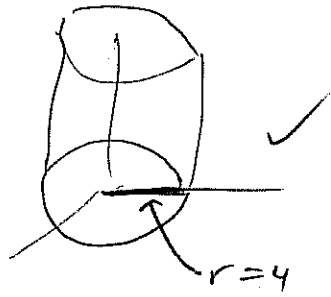
$= P$ $= Q$ $= R$

False. Consider the xy-, xz- and yz-planes.

P and Q are both orthogonal to R, but $P \perp Q$, not parallel. ✓

(d) **True or False:** In cylindrical coordinates, the equation $r = 4$ is a cylinder.

True. ✓



2. (8 points) Let $\mathbf{u} = \langle 0, 2, 3 \rangle$, $\mathbf{v} = \langle -1, 0, 4 \rangle$ and $\mathbf{w} = \langle 2, -3, 0 \rangle$. Calculate the following expressions. If the expression does not exist or does not make sense, explain why.

(a) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$

Does not exist s/c $u \cdot v$ is a scalar, and you cannot cross a scalar and a vector.

2

(b) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

$$u \times v = \begin{matrix} i & j & k & i & j \\ 0 & 2 & 3 & 0 & 2 \\ -1 & 0 & 4 & -1 & 0 \end{matrix}$$

4

$$\vec{u} \times \vec{v} = 8\vec{i} - 3\vec{j} + 0\vec{k} - (-2\vec{k}) - 0\vec{i} - 0\vec{j} = \langle 8, -3, 2 \rangle$$

$$\langle 8, -3, 2 \rangle \cdot \langle 2, -3, 0 \rangle = 16 + 9 + 0 = \boxed{25}$$

2

P Q

3. (6 points) Find the equation of the plane that goes through the points $(0,0,8)$, $(0,2,0)$ and $(3,0,0)$.

R

$\vec{PQ} = \langle 0, 2, -8 \rangle$ ✓

$\vec{PR} = \langle 3, 0, -8 \rangle$ ✓

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -8 \\ 3 & 0 & -8 \end{vmatrix}$$ ✓ ✓ ✓

$$= -16\vec{i} - 24\vec{j} - 0\vec{k} - 0\vec{i} - 0\vec{j} - 6\vec{k} = \langle -16, -24, -6 \rangle$$

Plane is $-16(x-0) - 24(y-0) - 6(z-8) = 0$

$-16x - 24y - 6z + 48 = 0$ ✓

4. (2 points) Write a set of parametric equations that represent the y-axis.

$x = 0$

$y = t$

$z = 0$

5. (6 points) x y z

(a) Convert the rectangular coordinates $(2\sqrt{2}, -2\sqrt{2}, 5)$ to cylindrical coordinates.

$r^2 = x^2+y^2 = 8+8 \Rightarrow r^2 = 16 \Rightarrow r = 4 \quad (\text{or } -4)$

3 $\tan(\theta) = \frac{y}{x} = -1 \Rightarrow \theta = \tan^{-1}(-1) = \frac{-\pi}{4}$

$\Rightarrow$ coordinates are $\boxed{(4, \frac{-\pi}{4}, 5)}$ or $(-4, \frac{3\pi}{4}, 5)$

(b) Convert the equation $\phi = \pi/3$ (which is in spherical coordinates) to rectangular coordinates.

$\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

3 $\Rightarrow \cos\frac{\pi}{3} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\frac{1}{2} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\left(\sqrt{x^2+y^2+z^2} = 2z\right)^2$

$x^2+y^2+z^2 = 4z^2$

$\boxed{x^2+y^2-3z^2 = 0}$

Extra Credit(1 point) Find nonzero vectors u, v, and w such that $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ but $\mathbf{v} \neq \mathbf{w}$.

$u = \langle 1, 0, 0 \rangle$ $\quad v = \langle 0, 1, 0 \rangle$

$w = \langle 1, 1, 0 \rangle$

$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k}$

$u \times w = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \vec{k}$

No Calculator Name: Key

6. (5 points) Match the equation to the graph.

(iii) (a) $z = y^2$ Cylindrical surface, parabola

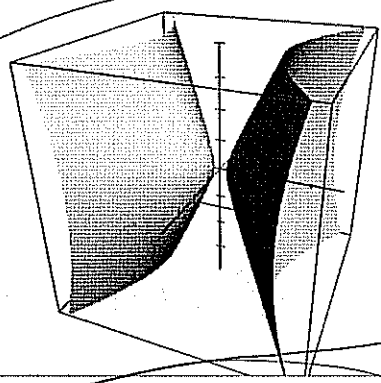
(i) (b) $4z^2 + 9x^2 - 16y^2 = -25$ $\Rightarrow -4z^2 - 9x^2 + 16y^2 = 25$ is Hyp. of 2 sheets

(iv) (c) $9z - 6x - 4y = 10$ is a plane

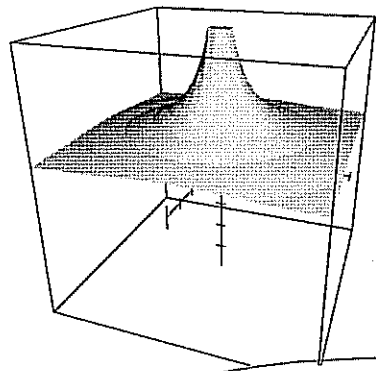
(vi) (d) $y^2 + x^2 = z^2 + 1$ $\Rightarrow y^2 + x^2 - z^2 = 1$ is hyp. of 1 sheet

(v) (e) $x^2 - y^2 - z = 0$ saddle

(b)

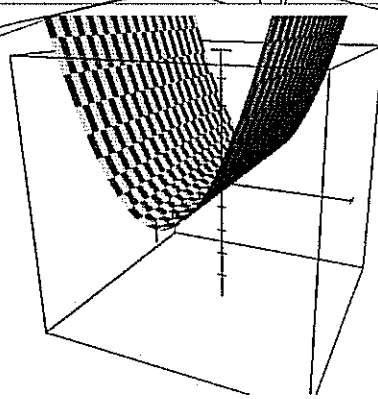


(i) _____

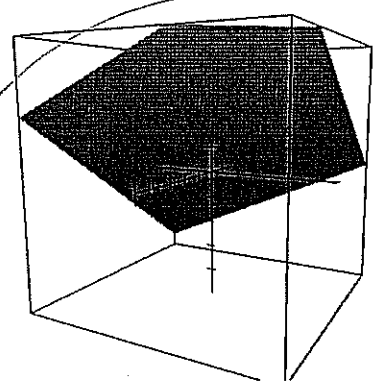


(ii)

(a)



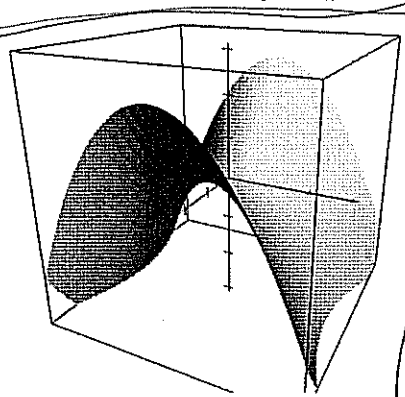
(iii)



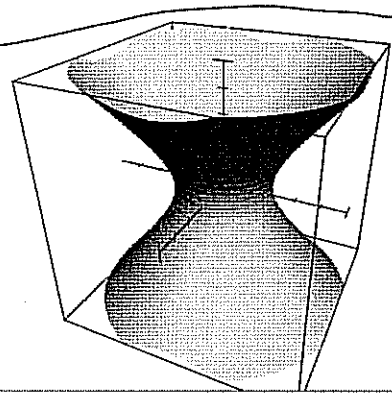
(iv)

(c)

(e)



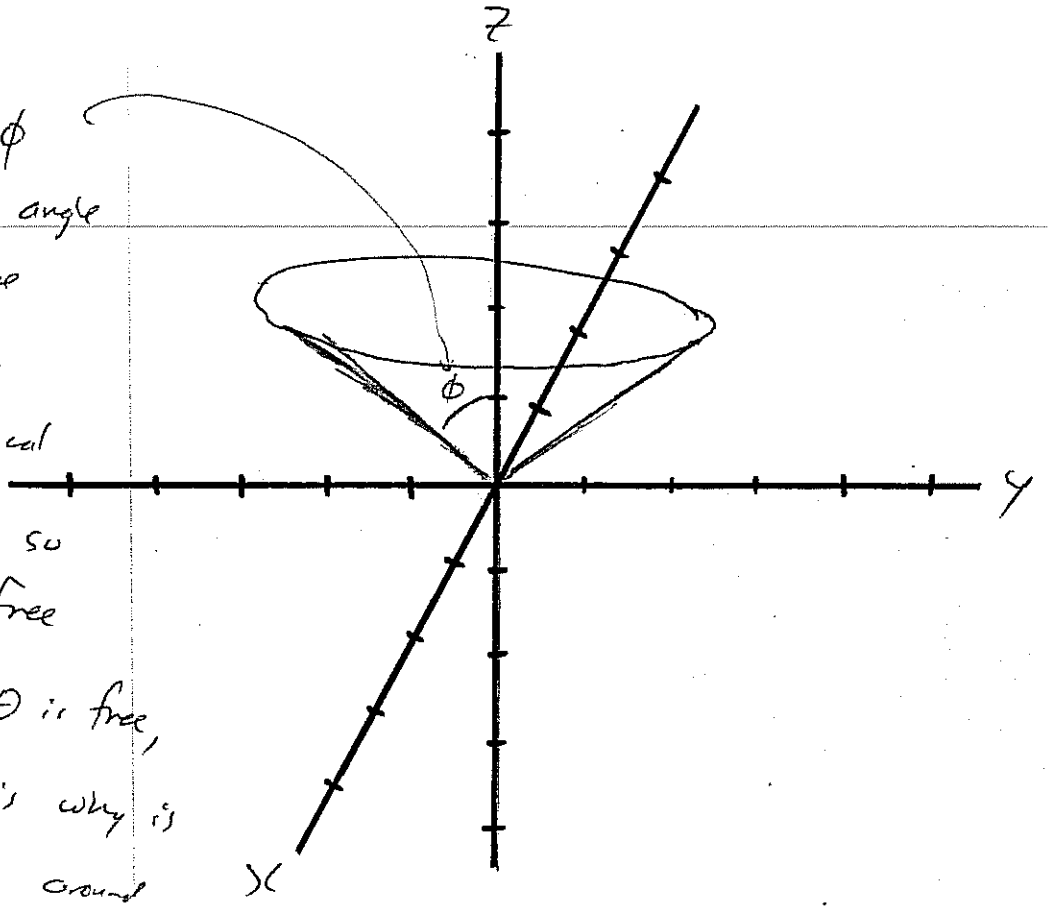
(v)



(vi) _____

(d)

7. (5 points) Sketch the surface given by the equation $\phi = \pi/3$ by hand. Explain in words how the equation corresponds to the graph.



angle ϕ measures angle from the z-axis. In spherical coordinates, so ρ is free and θ is free, this is why is rotates around the z-axis.

Since $\rho \geq 0$, only get top half of the cone.